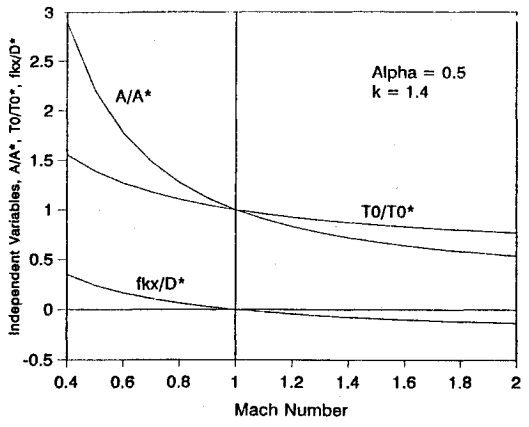

Fig. 1 Independent variable functions.

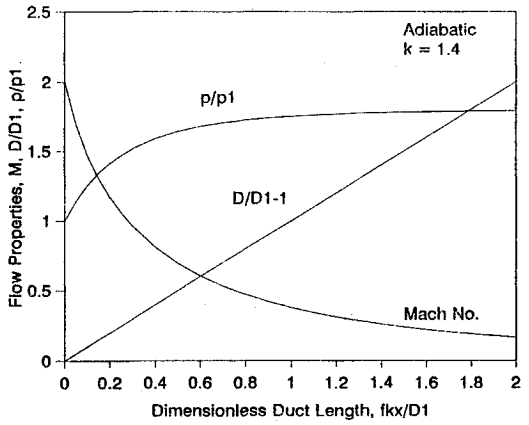

Fig. 2 Flow property relations.

The variations in duct diameter, Mach number, and static pressure are presented in Fig. 2 as a function of the dimensionless duct length. The flow is adiabatic ($\alpha = 1$), and the initial Mach number is 2. The diameter and pressure have been normalized to 1 at the entrance. Figure 2 shows that the duct is a divergent conic. Figure 2 also shows that the transition from a supersonic to a subsonic Mach number is continuous and without a shock.

From the relationship between the duct diameter and length in Fig. 2 and Eq. (2), the divergent angle ϕ can be determined:

$$(D/D_1 - 1)/2x/D_1 = (fkx/D1)/(2\alpha x/D_1) = \tan\phi$$

or

$$f = (2\alpha/k)\tan\phi \qquad (12)$$

For adiabatic flow ($\alpha = 1$) and the typical divergent angle of 15 deg, the value for f is 0.383. This is a relatively high value for the f and is indicative of laminar flow with a Reynolds number in the order of 1×10^2. This range of Reynolds number agrees well with the shockless transitions obtained in Refs. 1 and 2.

Conclusions

The analysis presents a theoretical explanation for shockless transition from supersonic to subsonic that has been observed experimentally and numerically. The analysis shows that shockless transition can only occur with the change in two independent variables. This is realistic since friction is always present. The analysis is based on generalized one-dimensional flow, which is somewhat limiting. Nevertheless, generalized one-dimensional flow results do provide an explanation for anomalies and guidance to more complex studies.

The analysis demonstrates shockless transition by removing the singularity that exists at Mach 1. This results in a set of "Mach functions" for the independent and dependent variables, which describe the direction of the transition. The functions indicate that transition from subsonic to supersonic flow is possible in convergent ducts without a throat under certain conditions.

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Numerical Studies on Droplet Breakup Models

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Introduction

ONE of the important aspects in spray combustion modeling is the dense-spray effects that include atomization process, drop breakup, droplet collision, and coalescence. Atomization process occurs on time and length scales too short to be resolved with practical computational grid sizes and time steps. Thus, atomization should be modeled as a sub-grid-scale process. To account for the dense spray effects, the present study employs an existing drop collision and coalescence model[1] and two breakup models, which are the Taylor analogy breakup (TAB) model[2] and the Reitz's wave instability model.[3] In the drop collision model, the probability distributions governing the number and outcomes of the collisions between two drops are sampled randomly in consistency with the stochastic particle tracking method. Both breakup models are based on the assumptions that atomization and drop breakup are indistinguishable processes within a dense spray near the nozzle exit. Accordingly, atomization is prescribed by injecting drops that have a characteristic size equal to the nozzle exit diameter. The present study is mainly motivated to evaluate the performance of these two droplet

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breakup models. With Eulerian-Lagrangian formulations in multiphase flows, the stochastic separated flow (SSF) approach[4,5] categorized in the discrete droplet model is employed to account for the turbulence effects on interphase transport. The k-ε model is used to characterize the time and length scales of the gas phase turbulence for droplet dispersions. Validation cases include the nonevaporating and evaporating solid-cone dense sprays.

Physical and Numerical Models

The gas-phase equation is written in an Eulerian coordinate, whereas the liquid-phase is presented in Lagrangian coordinates. The two-way coupling between the two phases is described by the interaction source terms that represent the rates of momentum, mass, and heat transfer. The governing equations and the detailed expressions of source terms can be found in Refs. 4 and 5. The drop collision model suggested by O'Rourke[1] is employed to calculate collision and coalescence among the dispersed liquid phase. The transient droplet vaporization process is represented by the infinitive conductivity model.[6]

Droplet Breakup

The present study employs two breakup models: 1) the TAB model[2] and 2) the Reitz's wave instability model.[3] The TAB model is based on an analogy between an oscillating and distorting droplet and a spring-mass system. The restoring force of the spring is analogous to the surface tension forces. The external force on the mass is analogous to the gas aerodynamic force. The damping forces due to liquid viscosity are introduced to this analogy. Compared to Reitz's model,[3] the TAB model has several advantages in terms of no need to input the spray angle, an easy introduction of liquid viscosity effects, and the explicit information of distortion and oscillation effects on the interphase exchange rates of mass, momentum, and energy. The major limitation of the TAB model is that only one oscillation mode can be tracked. The droplet oscillation and breakup calculations require additional information for deformation and oscillation that can be determined by the equation for the acceleration of the droplet distortion parameter. Occurrence of droplet breakup, the Sauter mean radius (SMR), and oscillation velocity for the product drop depend on these two parameters and the Weber number. The radius of the product drops is then chosen randomly from a chi-squared distribution with calculated SMR. Following breakup, the product drop has the same temperature with the parent drop, and its deformation and oscillating parameters are set to zero.

In Reitz's wave instability model,[3] the primary and secondary breakup is modeled using a linear stability analysis for liquid jets. This method is capable of predicting the intact core length as well as various regimes of breakup due to the action of different combinations of liquid inertia, surface tension, and aerodynamic forces on the jet. The limitations include one characteristic size dimension for "blob" and inability to predict drop size distribution and time between breakups. In this model, the mean drop size and drop breakup rate are given by

$$r = \begin{cases} B_0\Lambda & \text{if} \quad B_0\Lambda \le a \\ \min \begin{cases} (3\pi a^2 W/2\Omega)^{0.33} \\ (3a^2\Lambda/4)^{0.33} \end{cases} & \text{if} \quad B_0\Lambda > a \end{cases} \tag{1}$$

$$\frac{da}{dt} = -\frac{(a - r)}{\tau_b} \tag{2}$$

where

$$\tau_b = 3.726 B_1 a/\Lambda\Omega \tag{3}$$

Here, B_0 is 0.61, B_1 is the breakup time constant and a is the radius of the liquid jet or the blob. The wavelength of the fastest growing wave Λ and the maximum wave growth rate Ω can be determined by the curve-fitted formulas that are obtained from the numerical solutions of the surface wave dispersion equation of a round jet. The secondary breakup is assumed to be governed by the same equations for the primary jet breakup. The finer drop parcel is generated when its mass reaches 20% of the parent drop mass. The breakup constant $B_1 = 10$, suggested by Reitz,[3] is used for atomization process, and $B_1 = 1.73$, suggested by O'Rourke and Amsden,[2] is employed for droplet secondary breakup.

Solution Procedure

The governing equations of the gas phase are solved using a finite volume method. Spatial differences are formed on a curvilinear general coordinate with all gas field variables stored at the same grid point. Second-order accurate central differencing scheme is used for the diffusion terms, and a second-order upwind scheme is used for the convection terms. The implicitly coupled pressure and velocity equations are solved by the improved PISO algorithm.[7] The strong coupling terms between particle and gas are evaluated by the same time-splitting technique. Implicit coupling procedures are used to treat momentum exchanges to avoid the small time steps. Accurate calculation of mass and heat transfer is achieved by automatic reductions in the timestep when the exchange rate becomes large.

Results and Discussion

The nonevaporating solid-cone spray measurements of Hiroyasu and Kadota[8] were used to validate the present numerical dense-spray models. The test conditions are given in Table 1. Liquid fuel is injected through a single nozzle into constant pressure, room-temperature nitrogen. The nozzle diameter is 0.30 mm. A computational domain of 20 mm in radius and 120 mm in length was discretized by a 25-radial and 45-axial grid. The mesh spacing was nonuniform with refinement on the centerline and close to the injector. The number of computational parcels used in calculations was about 2000. The initial turbulent quantities were assumed as the small values ($k = 1 \times 10^{-3}$ m^2/s^2, $\varepsilon = 4 \times 10^{-4}$ m^2/s^3. The numerical results were insensitive to these initial values.

Figure 1a shows the comparison of predicted and measured spray tip penetration. The calculated penetration with two breakup models show reasonably good agreement with the measurement. Compared to the TAB model,[2] the Reitz's model[3] slightly overpredicts the penetration length. However, the discrepancies in the penetration length could be partially attributed to the imprecise definition of the spray tip. In the present computations, the spray tip was defined to be the location of the leading spray drop parcel. Figure 1b shows the variation of SMD for two breakup models. The three data at 65 mm correspond to the measurements. The computed drop size is time-averaged over the spray cross section at each axial location. The overall trend of the SMD distribution is similarly predicted by two models. Close to the injector, the drop size decreases rapidly due to drop breakup. Further downstream, the drop size increases gradually due to drop coalescence. The predicted drop sizes at 65 mm are qualitatively agreed with the experimental data for this case. The

Table 1 Test conditions for nonevaporating and evaporating spray

Case	P_{inj}, MPa	P_{gas}, MPa	T, K	M_{inj}, kg/s	Atmosphere
Nonevaporating spray	9.9	1.1	300	0.00688	N_2
Evaporating spray	30	3.0	900	0.00326	N_2

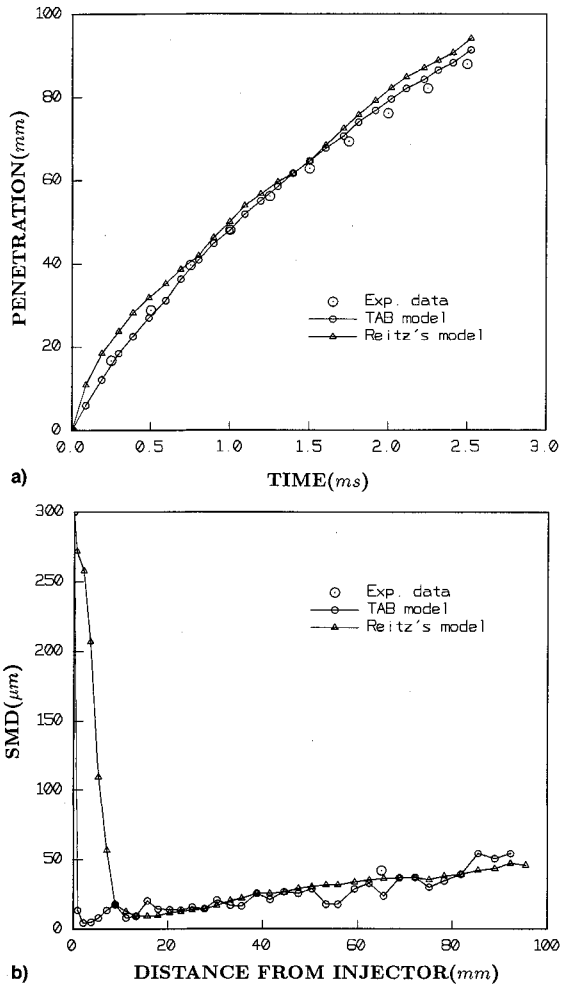

Fig. 1 a) Spray tip penetration vs time and b) SMD vs axial distance.

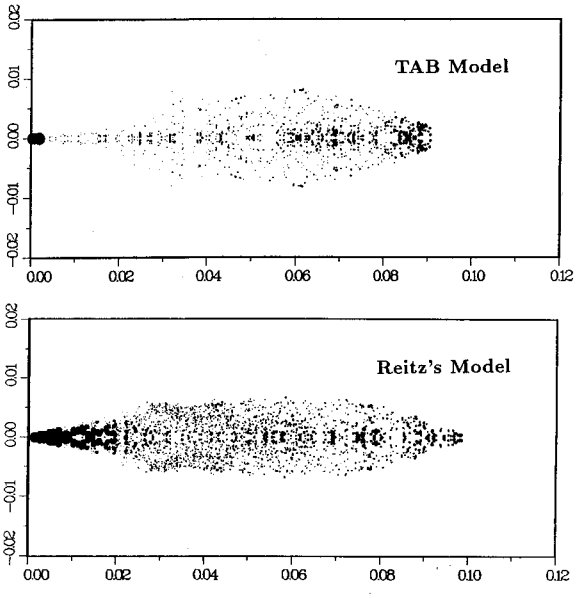

Fig. 2 Spray parcel distribution in a nonevaporating spray.

discrepancy could be partly associated to the fact that the experimental sprays were pulsed while the computations assumed a constant pressure injection for the entire computational time period. In comparison with the Reitz's model, the TAB model provides unreasonably rapid breakup rate near the injector. Especially close to the injector, the TAB model predicts a much faster breakup than the Reitz's model. Due to this faster breakup rate near the injector, the TAB model

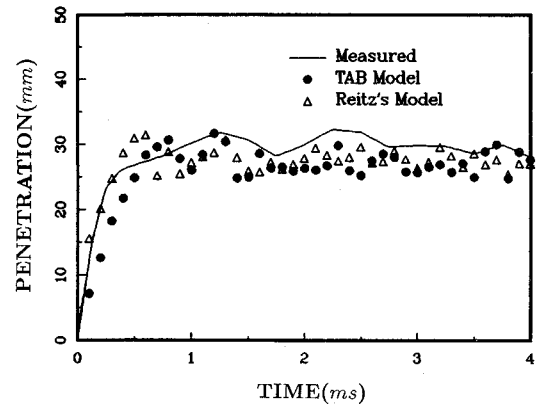

Fig. 3 Spray tip penetration vs time in an evaporating spray.

significantly underpredicts the intact core length. As shown in Fig. 2, the predicted intact core lengths (20 mm for P = 1.1 MPa) have a reasonable agreement with the empirical jet length correlation suggested by Arai et al.[9] These results indicate that the Reitz's wave instability model is capable of predicting the intact core length with reasonable accuracy.

To validate the present model in the evaporating spray situations, the measurements of Yokoda et al.[10] have been selected. The test conditions are described in Table 1. Liquid fuel is injected through a single hole nozzle into high-pressure, high-temperature nitrogen. The nozzle diameter was 0.16 mm. A computational domain of 20 mm in radius and 100 mm in length was discretized by a 21-radial and 44-axial grid.

In this evaporating solid-cone spray, we focus on the comparative performance of the TAB breakup model[2] and the Reitz's wave instability breakup model.[3] Comparisons of the computed and experimental spray penetration vs time are shown in Fig. 3. Two breakup models yield an overall agreement with experimental data of spray penetration. However, at the initial stage of the injection, the TAB model noticeably underpredicts the penetration length. Again, the underpredicted penetration length with the TAB model results from the rapid breakup rate near the injector. These numerical results suggest that the Reitz's breakup model,[3] in comparison with the TAB model, is more suitable for predicting the complex dense spray dynamics.

Summary

In comparison with the TAB model, the Reitz's wave instability model reasonably well-predicts the intact core length of the nonevaporating spray. At the initial stage of the injection, the TAB model noticeably underpredicts the penetration length of evaporating spray due to the relatively rapid breakup rate near the injector. To improve the numerical modeling of predicting the primary breakup regime, the liquid displacement effect within the computational cell should be considered.

Acknowledgments

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